10th Class 2020

Math (Science)	Group-I	PAPER-II
Time: 2.10 Hours	(Subjective Type)	Max. Marks: 60

(Part-I)

2. Write short answers to any SIX (6) questions: (12)

(i) Define exponential equation and give an example.

Ans In an equation, if variable occurs in exponent, then it is called exponential equation.

For example, $5^{1+x} + 5^{1-x} = 26$.

(ii) Solve: $(2x - \frac{1}{2})^2 = \frac{9}{4}$.

Ans Given: $(2x - \frac{1}{2})^2 = \frac{9}{4}$

By taking under root both sides, we get

$$\sqrt{(2x - \frac{1}{2})^2} = \sqrt{\frac{9}{4}}$$

$$2x - \frac{1}{2} = \pm\frac{3}{2}$$

$$2x - \frac{1}{2} = \frac{3}{2} \quad ; \quad 2x - \frac{1}{2} = \frac{-3}{2}$$

$$2x = \frac{3}{2} + \frac{1}{2} \quad ; \quad 2x = \frac{-3}{2} + \frac{1}{2}$$

$$= \frac{3+1}{2} \quad ; \quad = \frac{-3+1}{2}$$

$$= \frac{4}{2} \quad ; \quad = \frac{-2}{2}$$

$$2x = 2 \quad ; \quad 2x = -1$$

$$\boxed{x = 1} \quad ; \quad \boxed{x = \frac{-1}{2}}$$

The solution set will be: $\left\{1, \frac{-1}{2}\right\}$.

(iii) **Solve the given equation using quadratic formula:**

$$2 - x^2 = 7x$$

Ans Given quadratic equation:

$$2 - x^2 = 7x$$

By arranging the above equation as standard form:

$$0 = 7x + x^2 - 2$$

$$\Rightarrow x^2 + 7x - 2 = 0$$

From above equation,

$$a = 1, b = 7, c = -2$$

The Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-7 \pm \sqrt{(7)^2 - 4(1)(-2)}}{2(1)}$$

$$= \frac{-7 \pm \sqrt{49 + 8}}{2}$$

$$= \frac{-7 \pm \sqrt{57}}{2}$$

The solution set will be:

$$\left\{\frac{-7 \pm \sqrt{57}}{2}\right\}$$

(iv) **Evaluate:** $(1 - \omega + \omega^2)^6$

Ans Given: $(1 - \omega + \omega^2)^6$

The above expression can be written as:

$$(1 + \omega^2 - \omega)^6$$

So, $(1 + \omega^2 - \omega)^6 = (-\omega - \omega)^6$ $\quad (\because \; 1 + \omega^2 = -\omega)$

$$= (-2\omega)^6$$

$$= (-2)^6 \omega^6$$

$$= 64 (\omega^3)^2$$

$$= 64 (1)^2 \quad (\because \; \omega^3 = 1)$$

$$= 64$$

(v) Using synthetic division, show that $x - 2$ is a factor of $x^3 + x^2 - 7x + 2$.

Ans $P(x) = x^3 + x^2 - 7x + 2$

And $x - a = x - 2$

So, $a = 2$

By synthetic division,

$$\begin{array}{c|cccc} & 1 & 1 & -7 & 2 \\ 2 & \downarrow & 2 & 6 & -2 \\ \hline & 1 & 3 & -1 & \multicolumn{1}{|c}{0} \end{array}$$

(vi) Write the quadratic equation having roots: 0, –3.

Ans Sum of the roots $= S = 0 + (-3)$

$= 0 - 3$

$= -3$

Product of the roots $= P = 0(-3)$

$= 0$

The standard form of quadratic equation, having roots, is:

$$x^2 - Sx + P = 0$$

By putting the values of S and P, we get

$$x^2 - (-3)x + 0 = 0$$

$$x^2 + 3x = 0 \quad \text{(Required Equation).}$$

(vii) Define proportion.

Ans A proportion is a statement, which is expressed as an equivalence of two ratios. If two ratios a : b and c : d are equal, then we can write a : b :: c : d; where quantities a, d are called extremes, while b, c are called means.

(viii) If $w \propto \frac{1}{v^2}$ and $w = 2$ when $v = 3$, then find w.

Ans Given,

$$w \propto \frac{1}{v^2}$$

$$\Rightarrow \quad w = \frac{k}{v^2} \qquad \text{(i)}$$

By putting $w = 2$ and $v = 3$ in equation (i), we get

$$2 = \frac{k}{(3)^2}$$

$$2 = \frac{k}{9}$$

$$\Rightarrow \quad \boxed{k = 18}$$

Now putting k = 18 in equation (i),

$$w = \frac{18}{(3)^2}$$

$$w = \frac{18}{9}$$

$$\boxed{w = 2}.$$

(ix) Find a third proportional to: $a^2 - b^2, a - b$.

Ans Let x = third proportional

$$a^2 - b^2 : (a - b) :: (a - b) : x$$

Product of Extremes = Product of Means

$$x(a^2 - b^2) = (a - b)(a - b)$$

$$x = \frac{(a - b)(a - b)}{(a^2 - b^2)}$$

$$x = \frac{(a - b)(a - b)}{(a + b)(a - b)}$$

$$x = \frac{a - b}{a + b}$$

Thus, the third proportional is $\frac{a - b}{a + b}$.

3. Write short answers to any SIX (6) questions: (12)

(i) Define improper fraction with an example.

Ans A rational fraction $\frac{N(x)}{D(x)}$, with $D(x) \neq 0$ is called an improper fraction, if degree of the polynomial N(x) is greater or equal to the degree of the polynomial D(x).

For example;

$$\frac{5x}{x+2}, \frac{3x^2+2}{x^2+7x+12}, \frac{6x^4}{x^3+1}$$

are improper fractions.

(ii) Resolve $\frac{5x+4}{(x-4)(x+2)}$ into partial fraction.

Ans Let $\frac{5x+4}{(x-4)(x+2)} = \frac{A}{x-4} + \frac{B}{x+2}$ (i)

Multiplying throughout by $(x-4)(x+2)$, we get

$$5x + 4 = A(x+2) + B(x-4) \quad \text{(ii)}$$

Equation (ii) is an identity, which holds good for all values of x and hence for

$$x = 4 \quad \text{and} \quad x = -2$$

Put $x - 4 = 0$ *i.e.*, $x = 4$ (factor corresponding to A) on both sides of the equation (ii), we get

$$5(4) + 4 = A(4+2)$$

$$20 + 4 = A(6)$$

$$\frac{24}{6} = A$$

$$\Rightarrow \quad \boxed{A = 4}$$

Put $x + 2 = 0$ *i.e.*, $x = -2$ (factor corresponding to B), we get

$$5(-2) + 4 = B(-2-4)$$

$$-10 + 4 = B(-6)$$

$$\Rightarrow \quad -6B = -6$$

$$\Rightarrow \quad \boxed{B = 1}$$

Thus, required partial fractions are $\frac{4}{x-4} + \frac{1}{x+2}$.

Hence, $\frac{5x+4}{(x-4)(x+2)} = \frac{4}{x-4} + \frac{1}{x+2}$.

(iii) If $X = \phi$, $Y = Z^+$, then find $X \cap Y$.

Ans Given,

$$X \cap Y = \phi \cap \{0, 1, 2, 3, ---\}$$

$$= \phi$$

(iv) **Find a and b, if (2a + 5, 3) = (7, b – 4).**

Ans By comparing the values, we get

$2a + 5 = 7$; $3 = b - 4$

$2a = 7 - 5$; $3 + 4 = b$

$2a = 2$; $7 = b$

$a = \frac{2}{2}$; ⇒ $\boxed{b = 7}$

$\boxed{a = 1}$

Thus, {a = 1, b = 7}.

(v) **If set M has 5 elements, then find the numbers of binary relations in M.**

Ans Number of elements in M = 5

Number of binary relations in M = $2^{5 \times 5}$

$= 2^{25}$

(vi) **Define a bijective function.**

Ans A function f : A → B is called bijective function 'f' is one-one and onto. For example, if A = {1, 2, 3, 4} and B = {2, 3, 4, 5}.

(vii) **The marks of seven students in Mathematics are as follows, calculate the arithmetic mean:**

Student No.	1	2	3	4	5	6	7
Marks	45	60	74	58	65	63	49

Ans

Student No.	Marks (X)
1	45
2	60
3	74
4	58
5	65
6	63
7	49
	414

For arithmetic Mean ($\bar{X}$):

$\bar{X} = \frac{\Sigma X}{n}$

$= \frac{414}{7}$

$\boxed{\bar{X} = 59.14}$

(viii) **Find the modal size of shoes for the following data:**
4, 4, 5, 5, 6, 6, 6, 7, 7, 5, 7, 5, 8, 8, 8, 6, 5, 6, 5, 7

Ans As the number '5' is repeated most of the times in the given data, so the modal size of shoes is 5.

(ix) **Define median and write its formula.**

Ans **Median:**

Median is the middle most observation in an arranged data set. It divides the data set into two equal parts. $\tilde{X}$ is used to represent median. We can determine median by using the following formulas:

Case-1:

When number of observation is odd:

$$\tilde{X} = \text{size of } \left(\frac{n+1}{2}\right) \text{th value.}$$

Case-2:

When number of observations is even:

$$\tilde{X} = \frac{1}{2}\left[\text{size of } \left(\frac{n}{2}\text{ th} + \frac{n+1}{2}\text{th}\right) \text{ value}\right].$$

4. Write short answers to any SIX (6) questions: (12)

(i) **Convert $\frac{3\pi}{4}$ radians to degrees.**

Ans
$$\frac{3\pi}{4} = \frac{3\pi}{4} \text{ Radian}$$
$$= \frac{3\pi}{4} \times 1 \quad \text{Radian}$$
$$= \frac{3\pi}{4} \times \frac{180^\circ}{\pi}$$
$$= 135^\circ$$

(ii) **Find 'r', when $l = 52$ cm, $\theta = 45^\circ$.**

Ans As we know that,

$$\theta = 45 \times \frac{\pi}{180} = \frac{\pi}{4}$$

$$r = \frac{l}{\theta}$$

$$= l \div \theta$$

$$= 52 \div \frac{\pi}{4}$$

$$= 52 \times \frac{4}{\pi}$$

$$= 52 \times 1.273$$

$$\boxed{r = 66.20}$$

(iii) In a ΔABC, a = 17 cm, b = 15 cm and c = 8 cm, find m∠A.

Ans By Pythagora's Theorem:

$$a^2 = c^2 + b^2$$

$$(17)^2 = (8)^2 + (15)^2$$

$$289 = 64 + 225$$

$$289 = 289$$

(iv) Define diameter of a circle.

Ans A chord which passes through the centre of the circle is called diameter of a circle.

(v) Define secant of a circle.

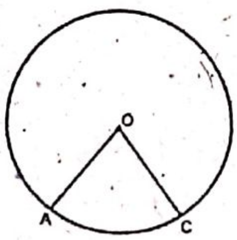
Ans A secant is a straight line which cuts the circumference of a circle in two distinct points.

(vi) Define circumference of the circle.

Ans The length of the boundary of the circle is called the circumference. It is calculated by $2\pi r$.

(vii) Define central angle of a circle.

Ans ∠AOC is the central angle of the circle whose vertex is at the centre O and its arms meet at the end points of arc $\overset{\frown}{AC}$.



(viii) Define circum circle.

Ans The circle passing through the vertices of triangle ABC is known as circum circle. Its radius as circum radius and centre as circum centre.

(ix) The length of each side of a regular octagon is 3 cm. Measure its perimeter.

Ans Length of side = 3 cm

Number of sides of an octagon = 8

Perimeter = Length × sides

$= 3 \times 8$

$= 24$ cm

(Part-II)

NOTE: Attempt THREE (3) questions in all. But question No. 9 is Compulsory.

Q.5.(a) Solve the equation: $2x^4 = 9x^2 - 4$. **(4)**

Ans For Answer see Paper 2019 (Group-I), Q.5.(a).

(b) Solve by using synthetic division if –1 is the root of the equation $4x^3 - x^2 - 11x - 6 = 0$. **(4)**

Ans Since –1 is the root of the equation:

$$4x^3 - x^2 - 11x - 6 = 0$$

Then by synthetic division, we get

	4	–1	–11	–6
–1	↓	–4	5	6
	4	–5	–6	0

The depressed equation is:

$$4x^2 - 5x - 6 = 0$$

$$4x^2 - 8x + 3x - 6 = 0$$

$$4x(x-2) + 3(x-2) = 0$$

$$(4x+3)(x-2) = 0$$

$$4x + 3 = 0 \quad ; \quad x - 2 = 0$$

$$4x = -3 \quad ; \quad x = 2$$

$$x = \frac{-3}{4}$$

Hence, $\frac{-3}{4}$, 2 and -1 are the roots of the given equation.

Q.6.(a) If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$ (a, b, c, d, e, f $\neq$ 0), then show that

$$\frac{ac+ce+ea}{bd+df+fb}=\left[\frac{ace}{bdf}\right]^{2/3}.$$ (4)

Ans Let $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=k$

$\Rightarrow$ $\frac{a}{b}=k$ and $\frac{c}{d}=k$ and $\frac{e}{f}=k$

$\Rightarrow$ $a=bk$ and $c=dk$ and $e=fk$

$$\text{L.H.S}=\frac{ac+ce+ea}{bd+df+fb}$$

$$=\frac{(bk)(dk)+(dk)(fk)+(fk)(bk)}{bd+df+fb}$$

$$=\frac{bd\,k^2+df\,k^2+fb\,k^2}{bd+df+fb}$$

$$=\frac{k^2(bd+df+bf)}{bd+df+bf}$$

$$=k^2 \quad \text{(i)}$$

$$\text{R.H.S}=\left[\frac{a\,c\,e}{b\,d\,f}\right]^{2/3}$$

$$=\left[\frac{(bk)(dk)(fk)}{b\,d\,f}\right]^{2/3}$$

$$=\left[\frac{bd\,fk^3}{b\,d\,f}\right]^{2/3}$$

$$=(k^3)^{2/3}$$

$$=k^2 \quad \text{(ii)}$$

From (i) and (ii), we have

L.H.S = R.H.S

Hence, $\frac{ac+ce+ea}{bd+df+fb}=\left[\frac{a\,c\,e}{b\,d\,f}\right]^{2/3}$.

(b) **Resolve into partial fractions:** $\frac{3x+7}{(x^2+1)(x+3)}$. (4)

Ans

$$\frac{3x+7}{(x^2+1)(x+3)} = \frac{Ax+B}{x^2+1} + \frac{C}{x+3}$$

Multiplying throughout by $(x^2+1)(x+3)$, we get

$$3x+7 = (Ax+B)(x+3) + C(x^2+1) \quad \text{(i)}$$

$$3x+7 = Ax^2 + 3Ax + Bx + 3B + Cx^2 + C$$

$$3x+7 = Ax^2 + Cx^2 + 3Ax + Bx + 3B + C \quad \text{(ii)}$$

To find C:

We put $x + 3 = 0 \Rightarrow x = -3$ in equation (i), we get

$$3(-3) + 7 = (A(-3) + B)(-3+3) + C((-3)^2 + 1)$$

$$-9 + 7 = (-3A + B) + (0) + C(9 + 1)$$

$$-2 = 10C$$

$$10C = -2$$

Dividing throughout by '10', we get

$$C = \frac{-1}{5}$$

To find A and B:

Equating coefficient of x^2 and constants on both sides of eq (ii), we get

$$A + C = 0$$

$$A + \left(\frac{-1}{5}\right) = 0$$

$$A = \frac{1}{5}$$

And $3B + C = 7$

$$3B + \left(\frac{-1}{5}\right) = 7$$

$$3B = 7 + \frac{1}{5}$$

$$3B = \frac{35+1}{5}$$

$$3B = \frac{36}{5}$$

$$B = \frac{36}{5} \times \frac{1}{3}$$

$$B = \frac{12}{5}$$

Thus, required partial fractions are

$$\frac{\frac{1}{5}x + \frac{12}{5}}{x^2 + 1} + \frac{\frac{-1}{5}}{x + 3}$$

$$= \frac{\frac{x + 1^2}{5}}{x^2 + 1} - \frac{\frac{1}{5}}{x + 3}$$

$$= \frac{x + 12}{5(x^2 + 1)} - \frac{1}{5(x + 3)}$$

Hence, $\frac{3x - 7}{(x^2 + 1)(x + 3)} = \frac{x + 12}{5(x^2 + 1)} - \frac{1}{5(x + 3)}$.

Q.7.(a) If U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, A = {1, 3, 5, 7, 9} and B = {2, 3, 5, 7}, then verify $(A \cap B)' = A' \cup B'$. **(4)**

Ans For Answer see Paper 2019 (Group-I), Q.7.(a).

(b) The marks of six students in Mathematics are given, determine variance. **(4)**

Student	1	2	3	4	5	6
Marks	60	70	30	90	80	42

Ans For Answer see Paper 2017 (Group-II), Q.7.(b).

Q.8.(a) Verify the identity: **(4)**

$$\cos^4\theta - \sin^4\theta = \cos^2\theta - \sin^2\theta$$

Ans L.H.S $= \cos^4\theta - \sin^4\theta$

$= (\cos^2\theta)^2 - (\sin^2\theta)^2$

$= (\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta)$

$= 1(\cos^2\theta - \sin^2\theta)$

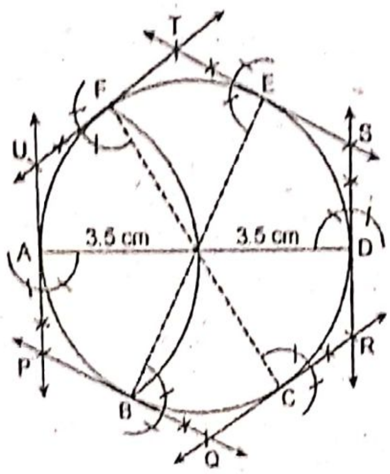
$= \cos^2\theta - \sin^2\theta$

= R.H.S

Proved.

(b) About a circle of radius 3.5 cm, describe a regular hexagon.

Ans

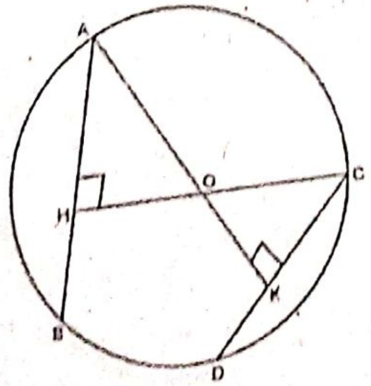


Steps of Construction:

1. Draw a diameter $\overline{AD}$ = 7 cm.
2. From point A draw an arc of radius $\overline{AO}$ = 3.5 cm (the radius of the circle), which cuts the circle at points B and F.
3. Join B with O and extend it to meet the circle at E.
4. Join F with O and extend it to meet the circle at C.
5. Draw tangents to the circle at points A, B, C, D, E and F intersecting one another at points P, Q, R, S, T and U, respectively.
6. Thus PQRSTU is the circumscribed regular hexagon.

Q.9. Prove that two chords of a circle which are equidistant from the centre are congruent. (8)

Ans



Given:

$\overline{AB}$ and $\overline{CD}$ are two chords of a circle with center at O. $\overline{OH} \perp \overline{AB}$ and $\overline{OK} \perp \overline{CD}$, so that $m\overline{OH} = m\overline{OK}$.

To Prove:

$m\overline{AB} = m\overline{CD}$

Construction:

Join A and C with O, so that we can form $\angle rt\ \Delta^s$ OAH and OCK.

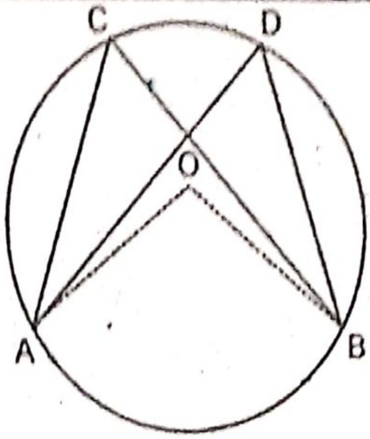
Proof:

Statements	Reasons
In $\angle rt\ \Delta^s$ OAH $\leftrightarrow$ OCK,	
$\because$ hyp. $\overline{OA}$ = hyp. $\overline{OC}$	Radii of the same circle
$m\overline{OH} = m\overline{OK}$	Given
$\therefore$ $\Delta OAH \cong \Delta OCK$	H.S postulate
So,	
$m\overline{AH} = m\overline{CK}$ (i)	Corresponding sides of congruent triangles
But	
$m\overline{AH} = \frac{1}{2} m\overline{AB}$ (ii)	$\overline{OH} \perp$ chord $\overline{AB}$ (Given)
Similarly,	
$m\overline{CK} = \frac{1}{2} m\overline{CD}$ (iii)	$\overline{OK} \perp$ chord $\overline{CD}$ Given
Since $m\overline{AH} = m\overline{CK}$	Already proved in (i)
$\therefore$ $\frac{1}{2} m\overline{AB} = \frac{1}{2} m\overline{CD}$	Using (ii) and (iii)
or $m\overline{AB} = m\overline{CD}$	

OR

Prove that any two angles in the same segment of a circle are equal.

Ans



Given:

m∠ACB = m∠ADB are the circumangles in the same segment of a circle with centre O.

To Prove:

m∠ACB = m∠ADB

Construction:

Join O with A and O with B.

So that ∠AOB is the central angle.

Proof:

Statements	Reasons
Standing on the same arc AB of a circle.	
∠AOB is the central angle whereas	Construction
∠ACB and ∠ADB are circumangles	Given
∴ m∠AOB = 2m∠ACB (i)	By theorem I (External angle is the sum internal opposite angle).
and m∠AOB = 2m∠ADB (ii)	By theorem I
⇒ 2m∠ACB = 2m∠ADB	Using (i) and (ii)
Hence, m∠ACB = m∠ADB	