Arbitrary Cash-Flow Valuation

An Arbitrary Cash-Flow (ACF) security interface values future known cash-flows. These cash-flows must be in a single (potentially foreign) currency. The present value of these cash-flows is determined by prevailing market interest and foreign exchange rates.

Suppose $C_1, C_2, ..., C_N$ denote known future cash-flows, which occur at the known times $T_1, T_2, ..., T_N$. If Df_T denotes the risk-free discount factor to time *T* for the cash-flow currency, then GET applies the following valuation formula to obtain the present value (PV) of the above cash-flows

$$\left[\sum_i Df_{T_i} \times C_i\right] \times Spot\ FX. \qquad (1)$$

Here, $Spot\ FX$ is the spot market foreign exchange (FX) rate from the cash-flow currency into the domestic currency.

$$\exp\left[-\int_0^t f(s)\,ds\right] = Df_t.$$

If $t_1, t_2, ..., t_m$ denote Imagine interest rate maturity dates for the simply compounded rates $R_1, R_2, ..., R_m$, then our benchmark assumed piece-wise constant continuously compounded instantaneous forward rates *f(t)* given by. In addition, the following is satisfied for all $t \geq 0$,

$$\exp\left[-\int_0^t f(s)\,ds\right] = \exp\left[-\left(\sum_{i=1}^{k-1} f_{t_i}(t_{i+1}-t_i) + f_k(t-t_k)\right)\right] =$$

$$\exp\left[-\left(\sum_{i=1}^{k-1} f_{t_i}(t_{i+1}-t_i)\right)\right] \times \exp\left[-f_k(t-t_k)\right] =$$

$$\frac{Df_{t_1}}{1}\frac{Df_{t_2}}{Df_{t_1}}\cdots\frac{Df_{t_{k-1}}}{Df_{t_{k-2}}}\frac{Df_{t_k}}{Df_{t_{k-1}}} \times \exp\left[-f_k(t-t_k)\right] =$$

$$Df_{t_k} \times \exp\left[\frac{t-t_k}{t_{k+1}-t_k}\ln\left[\frac{Df_{t_{k+1}}}{Df_{t_k}}\right]\right] =$$

$$= Df_{t_k} \times \left[\frac{Df_{t_{k+1}}}{Df_{t_k}}\right]^{\frac{t-t_k}{t_{k+1}-t_k}}.$$

Moreover, for $t \in (t_k, t_{k+1}]$, we have

We note that, in the above, we equivalently apply log-linear interpolation of discount factors.

In the case, when we considered an Imagine bond par yield curve input, we performed the following calculations:

Suppose $y_1, y_2, ..., y_m$ denoted annually compounded par yields, with respect to maturities $1\ year, 2\ year, ..., m\ year$, for bonds $B_1, B_2, ..., B_m$. Thus, bond B_i has an annual coupon equal to $y_{i\ year}$.

We performed a bootstrap to obtain the discount factors $Df_{1\ year}, Df_{2\ year}, ..., Df_{m\ year}$. In particular,

$$Df_{1\ year} = \frac{1}{1+y_{1\ year}},$$

$$Df_{i\ year} = \frac{1-\sum_{j=1}^{i-1} y_{j\ year} Df_{t_{j\ year}}}{1+y_{i\ year}},\ i>1.$$

we applied the following recursive formulas

The equations above are derived from the following observation; given a notional of 100

$$100 = present\ value\ bond\ cashflows = \sum_{j=1}^{i-1} 100 y_{j\ year} Df_{T_{j\ year}} + 100\left(1 + y_{i\ year}\right) Df_{i\ year}.$$

the price of the par yield bond B_i is 100. Thus,

Finally, we assume as above that continuously compounded instantaneous forward rates *f(t)* satisfy

$$f(t) = \frac{-1}{t_{i\ year+1} - t_{i\ year}} \ln\left[\frac{Df_{t_{i\ year+1}}}{Df_{t_{i\ year}}}\right],\ t \in \left(t_{i\ year}, t_{i\ year+1}\right]$$

$$f(t) = \frac{-1}{t_{1\ year}} \ln\left[Df_{t_{1\ year}}\right],\ t \in \left[0, t_{1\ year}\right],$$

that is, log-linear interpolation of discount factors.

The implementation requires the following input parameters:

- Future cash-flow amounts,
- Future cash-flow times,
- Cash-flow currency,
- Domestic currency,
- Spot FX rate from the cash-flow to the domestic currency,

- Imagine interest rate curve for the cash-flow currency.

For a rate curve, the user must specify the following:

- a series of interest rates with start and end dates,
- interest rate compounding,
- day count convention.

For a bond yield curve (ref https://finpricing.com/lib/FiZeroBond.html), the user must specify the following:

- a series of bond yields with maturity dates,
- bond yield compounding,
- day count convention,
- coupon frequency,
- coupon amounts (as percentage of notional amount).

For a discount curve, the user must specify a series of discount factors and discount dates.